Subcooled Forced-Convection Film Boiling in the Presence of a Pressure Gradient

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An integral procedure based on the two-phase boundary-layer theory has been proposed for the analysis of forced-convection film boiling on plane and axisymmetric bodies. The analysis assumes constant properties and a large liquid/vapor density ratio. Effects of the pressure gradient, vapor superheating, liquid subcooling, and liquid Prandtl number on the skin-friction and heat-transfer characteristics are investigated in detail. Asymptotic expressions, which include those obtained by Walsh and Wilson, have been reduced in consideration of possible limiting conditions. It has been found that, when the subcooling is small, the skin-friction coefficient grouping $C_{fx}\ Rex^{1/2}$ decreases toward its minimum level, then increases gradually, and subsequently overshoots the level of the single-phase flow as the superheating becomes so significant that a strong vapor flow acceleration takes place.

Nomenclature

C_{fx} = local skin-friction coefficient
C_p = specific heat at constant pressure
D = shape factor
f = function for velocity profile in a liquid layer
h_{fg} = latent heat of vaporization
H = sensible latent heat ratio
i = 1 for a wedge and 3 for a cone
k = thermal conductivity
m = Falkner-Skan parameter, $m^* = m/i$
Nux = local Nusselt number
Pr = Prandtl number
$\overset{*}{r}$ = 1 for a wedge and $x \sin\phi$ for a cone
R = density/viscosity ratio
Rex = Reynolds number, $u_e x/\nu_f$
T = temperature
ΔT = degree of superheating
ΔT_f = degree of subcooling
u, v = x and y direction velocity components
u_i = interfacial velocity, $u_i^* = u_i/u_e$
x, y = boundary-layer coordinates
δ = vapor film thickness
Δ = viscous boundary-layer thickness of liquid
Δ_t = thermal boundary-layer thickness of liquid
ζ, ζ^* = boundary-layer thickness ratio, $\zeta^* = \zeta/\sqrt{Pr_f}$
η, η_t = similarity variables
θ = function for temperature profile in a liquid layer
Γ = parameter associated with the degree of subcooling
Λ = shape factor for velocity profile in a vapor layer
μ, ν = viscosity and kinematic viscosity
ρ = density

Subscripts

e = boundary-layer edge
f = liquid layer
i = liquid/vapor interface
w = wall

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Introduction

WHEN an object that has been aerodynamically heated enters water, film boiling may arise due to the influence of a sufficiently high surface temperature. A vapor film covering a heated surface results in a significant reduction in the drag force. Thus, film boiling may be a possible means of reducing the drag experienced by bodies moving through liquids.

Forced-convection film boiling is also observed in the process leading to vapor explosion. A molten metal suddenly brought into contact with a liquid coolant becomes surrounded by a film of coolant vapor that limits the rate of heat release. The vapor explosion takes place through the transition to rapid heat release. Many other examples of film boiling may be found in practical engineering applications.

Cess and Sparrow[1,2] studied the subcooled forced convection film boiling on a flat plate, using a combined analytical-numerical method that was later extended by Ito and Nishikawa[3] to cover a wide range of subcooling and superheating. For practical situations, however, a streamwise pressure gradient will exist in the liquid layer. This pressure gradient will then be transmitted to the vapor layer, where its effect will be greatly amplified due to the large density ratio between the liquid and vapor layers. Bromley et al.[4] realized this fact and analyzed the forced-convection film boiling on a circular cylinder in a saturated liquid. A similar analysis was made by Kobayashi[5] for the case of a sphere. In most practical cases, the liquid will be subcooled. Walsh and Wilson[6] made an elegant analysis, carefully choosing dimensionless variables, and obtained asymptotic expressions for two limiting cases corresponding to small and large values of liquid subcooling. Although the asymptotic solutions obtained by Walsh and Wilson are correct, no extensive comparison was made between the asymptotic and numerical results. Thus, the ranges where these two asymptotic expressions are valid have not been examined in detail. Moreover, small liquid Prandtl numbers were assumed for the case of large subcooling.

It is the purpose of this paper to investigate the effects of the pressure gradient, vapor superheating, liquid subcooling, and liquid Prandtl number on the heat-transfer and hydrodynamic characteristics in the laminar forced-convection film boiling. Since it is not always possible to extract meaningful asymptotic behavior from a limited number of sets of numerical calculations, an integral approach rather than a numerical approach will be adopted here. The analysis may be regarded as an extended version of the integral analysis, successfully employed for the single-phase forced-[7] and free-convection problems[8] and, subsequently, for the forced-convection film boiling on a flat plate.[9] It is particularly suited for investigation of the combined effects of the parameters associated with the problems of forced-convection film boiling. A wedge with an arbitrary included angle will be treated for an illustrative purpose. Results can readily be translated to the axisymmetric case. General asymptotic expressions, which naturally include those by Walsh and Wilson[6] as two limiting cases, are reduced by considering all possible asymptotic conditions. It will be shown that, when the liquid subcooling is small, the skin-friction coefficient grouping $C_{fx}Rex^{1/2}$ initially decreases with the increasing degree of the vapor superheating, then increases, and eventually exceeds the level of the single-phase flow as the vapor superheating and vapor flow acceleration become significant.

Analysis

The physical model considered for study is shown in Fig. 1. A wedge (or a cone) heated to a constant temperature T_w is placed in a liquid flowing at its freestream velocity $u_e(x)$. The bulk of the liquid is subcooled at its temperature T_e below the interfacial (saturation) temperature T_i corresponding to the system pressure. The vapor film and liquid boundary layer are regarded as stable and smooth. A set of the governing equations in the boundary layer coordinates (x,y) runs as shown in the following equations.

Vapor:

$$\frac{\partial}{\partial x}\overset{*}{r}u+\frac{\partial}{\partial y}\overset{*}{r}v=0 \tag{1}$$

$$\nu\frac{\partial^2 u}{\partial y^2}+\frac{\rho_f}{\rho}u_e\frac{du_e}{dx}=0 \tag{2}$$

$$k\frac{\partial^2 T}{\partial y^2}=0 \tag{3}$$

Liquid:

$$\frac{\partial}{\partial x}\overset{*}{r}u+\frac{\partial}{\partial y}\overset{*}{r}v=0 \tag{4}$$

$$\rho_f u\frac{\partial u}{\partial x}+\rho_f v\frac{\partial u}{\partial y}=\mu_f\frac{\partial^2 u}{\partial y^2}+\rho_f u_e\frac{du_e}{dx} \tag{5}$$

$$\rho C_{pf}u\frac{\partial T}{\partial x}+\rho C_{pf}v\frac{\partial T}{\partial y}=k_f\frac{\partial^2 T}{\partial y^2} \tag{6}$$

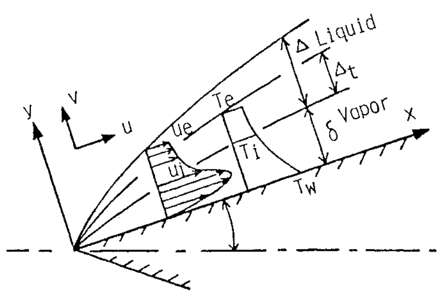

Fig. 1 Physical model and coordinates.

where

$$\overset{*}{r}=1 \qquad \text{for a wedge}$$

and

$$\overset{*}{r}=x\sin\phi \qquad \text{for a cone} \tag{7}$$

The boundary and compatibility conditions are at the wall surface $(y=0)$

$$u=v=0 \tag{8a}$$

$$T=T_w \tag{8b}$$

at the velocity/liquid interface $(y=\delta)$

$$u=(u)_f\equiv u_i \tag{9a}$$

$$\mu\frac{\partial u}{\partial y}=\left(\mu_f\frac{\partial u}{\partial y}\right)_f \tag{9b}$$

$$\rho u\frac{d\delta}{dx}-\rho v=\left(\rho u\frac{d\delta}{dx}-\rho v\right)_f \tag{9c}$$

$$T=(T)_f\equiv T_i \tag{9d}$$

$$\left(\rho u\frac{d\delta}{dx}-\rho v\right)h_{fg}=\left(k_f\frac{\partial T}{\partial y}\right)_f-k\frac{\partial T}{\partial y} \tag{9e†}$$

and in the bulk of fluid $(y=\infty)$

$$u=u_e(x) \tag{10a}$$

$$T=T_e \tag{10b}$$

where

$$u_e\propto x^m \tag{11a}$$

with, for a wedge

$$m=\pi/(\pi-\phi) \tag{11b}$$

or, for a cone

$$P'_{m+1}(-\cos\phi)=0 \tag{11c}$$

In Eq. (11c), the prime denotes differentiation and the function P_{m+1} is the Legendre function that remains finite when its argument equals unity for all values of its order $m+1$. A table for m and apex half-angle ϕ may be found in the work by Hess and Faulkner.[10] It is assumed that the properties of the liquid and vapor are constant. Properties associated with the liquid phase are identified by the subscript f, while those associated with the vapor phase are presented without subscripts (such as the specific heat C_p, density ρ, thermal conductivity k, viscosity μ, and kinematic viscosity ν). The subscripts w, i, and e refer to the wall, interface, and liquid boundary-layer edge, respectively. The continuity of the pressure at the liquid/vapor interface is already implemented in Eq. (2) as the term $(\rho_f/\rho)u_e(du_e/dx)$, which can be quite large due to large ρ_f/ρ. As seen in Eqs. (2) and (3), the inertia and convection terms in the vapor phase are dropped since the neglect of these terms results in very little loss of accuracy, as substantiated by Cess and Sparrow.[1] Equations (2) and (3) may readily be integrated with the boundary conditions given by Eqs. (8), (9a), and (9d) as

$$u/u_i=(1+\Lambda)(y/\delta)-\Lambda(y/\delta)^2 \tag{12a}$$

†The liquid's lateral convection away from the interface is neglected.

and

$$(T-T_i)/\Delta T = 1-(y/\delta) \tag{12b}$$

where

$$\Lambda = -\frac{1}{2}\frac{\delta^2}{u_i}\frac{\partial^2 u}{\partial y^2}\bigg|_{y=0} = \frac{m}{2}\frac{\rho_f}{\mu}\frac{u_e^2\delta^2}{u_i x} \tag{12c}$$

and

$$\Delta T = T_w - T_i \tag{12d}$$

Upon integrating Eq. (5) across the liquid layer with the aid of Eqs. (1), (4), (9c), and (10a), one obtains the following momentum integral relation:

$$\frac{\mathrm{d}}{\mathrm{d}x}\overset{*}{r}\int_\delta^{\delta+\Delta}(u_e u - u^2)\mathrm{d}y + \overset{*}{r}\frac{\mathrm{d}u_e}{\mathrm{d}x}\int_\delta^{\delta+\Delta}(u_e-u)\mathrm{d}y + \left(\frac{\rho}{\rho_f}\right)(u_e-u_i)\frac{\mathrm{d}}{\mathrm{d}x}\overset{*}{r}\int_0^\delta u\mathrm{d}y = \overset{*}{r}\nu_f\frac{\partial u}{\partial y}\bigg|_{y=\delta} \tag{13a}$$

where Δ is the viscous (velocity) boundary-layer thickness in the liquid layer. The third term on the left side which represents the momentum transfer to the vapor phase, is usually negligible since the density ratio ρ/ρ_f is extremely small. Hence,

$$\frac{\mathrm{d}}{\mathrm{d}x}\overset{*}{r}\int_\delta^{\delta+\Delta}(u_e u - u^2)\mathrm{d}y + \overset{*}{r}\frac{\mathrm{d}u_e}{\mathrm{d}x}\int_\delta^{\delta+\Delta}(u_e-u)\mathrm{d}y = \overset{*}{r}\nu_f\frac{\partial u}{\partial y}\bigg|_{y=\delta} \tag{13b}$$

An equivalent approximation was adopted by Cess and Sparrow[1] in their combined analytical-numerical analysis on a flat plate. In a similar fashion, the energy equation (6) can be integrated along with Eqs. (1), (9c), and (10b) to obtain

$$\frac{\mathrm{d}}{\mathrm{d}x}\overset{*}{r}\int_\delta^{\delta+\Delta_t}u(T-T_e)\mathrm{d}y + \left(\frac{\rho}{\rho_f}\right)\Delta T_f\frac{\mathrm{d}}{\mathrm{d}x}\overset{*}{r}\int_0^\delta u\mathrm{d}y = -\overset{*}{r}\frac{\nu_f}{Pr_f}\frac{\partial T}{\partial y}\bigg|_{y=\delta} \tag{14a}$$

where

$$\Delta T_f = T_i - T_e \tag{14b}$$

and where Pr_f is the liquid Prandtl number and Δ_t is the thermal boundary-layer thickness in a liquid layer, which may or may not exceed the viscous boundary-layer thickness Δ. The fact $\rho/\rho_f \ll 1$ is again utilized to simplify Eq. (14a) as

$$\frac{\mathrm{d}}{\mathrm{d}x}\overset{*}{r}\int_\delta^{\delta+\Delta_t}u(T-T_e)\mathrm{d}y = -\overset{*}{r}\frac{\nu_f}{Pr_f}\frac{\partial T}{\partial y}\bigg|_{y=\delta} \tag{14c}$$

Within the liquid layer, the following parabolic profiles are assumed to prevail:

$$u/u_e \equiv f(\eta;\ u_i^*) = u_i^* + (1-u_i^*)(2\eta-\eta^2) \tag{15a}$$

and

$$(T-T_e)/\Delta T_f \equiv \theta(\eta_t) = (1-\eta_t)^2 \tag{15b}$$

where

$$u_i^* \equiv u_i/u_e \tag{15c}$$

$$\eta \equiv (y-\delta)/\Delta \tag{15d}$$

and

$$\eta_t \equiv (y-\delta)/\Delta_t \tag{15e}$$

Upon substituting Eqs. (15a), (15b), (12a), and (12b) into Eqs. (13b), (14c), (9e), and (9b), differentiation and integration are carried out to obtain the following respective expressions for the viscous and thermal boundary-layer thicknesses in the liquid and the vapor film thickness:

$$\left(\frac{\Delta}{x}\right)^2 Rex\ i = \frac{30}{(1+3m^*)[1+(3/2)u_i^*]+5m^*} \tag{16}$$

$$\left(\frac{\Delta_t}{x}\right)^2 Rex\ i = \frac{4}{(1+m^*)DPr_f} \tag{17}$$

$$\left(\frac{\delta}{x}\right)^2 Rex\ i = \frac{\nu H}{\nu_f Pr}\frac{12}{(3+\Lambda)}\frac{1-[(1-\Lambda)/(1-u_i^*)]u_i^*\zeta^*\Gamma\sqrt{Pr_f}}{(1+m^*)u_i^*} \tag{18}$$

and

$$\frac{\delta}{\Delta} = \frac{\mu}{\mu_f}\frac{u_i^*}{2}\left(\frac{1-\Lambda}{1-u_i^*}\right) \tag{19}$$

where

$$Rex = u_e x/\nu_f \tag{20a}$$

$$H = C_p\Delta T/h_{fg} \tag{20b}$$

$$\zeta^* = \zeta/\sqrt{Pr_f} \tag{20c}$$

with

$$\zeta = \Delta/\Delta_t \tag{20d}$$

$$\Gamma = (Pr/Pr_f)(C_{pf}\Delta T_f/C_p\Delta T) \tag{20e}$$

$$m^* = m/i \tag{20f}$$

with

$$i = 1 \quad \text{for a wedge}$$

$$i = 3 \quad \text{for a cone} \tag{20g}$$

and

$$D(\zeta, u_i^*) \equiv \int_0^1 \theta(\eta_t) f[\min(\eta_t/\zeta, 1);\ u_i^*]\,\mathrm{d}\eta_t \tag{20h}$$

namely,

$$D = [(5\zeta-1)+(10\zeta^2-5\zeta+1)u_i^*]/30\zeta^2 \qquad \text{for } \zeta \geq 1$$

and

$$D = [(10-10\zeta+5\zeta^2-\zeta^3)+(10\zeta-5\zeta^2+\zeta^3)u_i^*]/30 \qquad \text{for } \zeta \leq 1 \tag{20i}$$

Rex is the Reynolds number based on ν_f, ζ the boundary layer thickness ratio in the liquid layer, ζ^* its modified form, H the sensible latent heat ratio related with the degree of vapor superheating, and Γ the parameter representing the degree of liquid subcooling. The modified wedge angle m^* is introduced with an integer i to present solutions once for both plane and axisymmetric cases. Thus, the results on wedges can readily be translated for those on cones.

Equations (16) and (17) are equated to give the following equation:

$$\zeta^{*2}=\frac{15}{2}\,\frac{(1+m^*)D(\zeta,u_i^*)}{(1+3m^*)[1+(3/2)u_i^*]+5m^*} \tag{21}$$

Then, Eqs. (16), (19), and (12c) are combined to yield the vapor/liquid viscosity ratio,

$$\frac{\mu}{\mu_f}=\frac{4}{15}\,\frac{\Lambda}{m^*u_i^*}\left(\frac{1-u_i^*}{1-\Lambda}\right)^2\left[(1+3m^*)\left(1+\frac{3}{2}u_i^*\right)+5m^*\right] \tag{22}$$

And lastly, Eqs. (18) together with Eq. (12c) gives

$$\left(\frac{H}{PrR}\right)\left(\frac{\mu}{\mu_f}\right)=\frac{1+m^*}{2m^*}\,\frac{u_i^{*2}\Lambda(1+\Lambda/3)}{1-[(1-\Lambda)/(1-u_i^*)]u_i^*\zeta^*(\Gamma\sqrt{Pr_f})} \tag{23}$$

where

$$R=\rho\mu/\rho_f\mu_f \tag{24}$$

Thus, the solution of the problem has finally been reduced to the determination of three unknown shape factors, namely, ζ^*, u_i^*, and Λ, for given parameters m^*, μ/μ_f, Pr_f, $\Gamma\sqrt{Pr_f}$, and H/PrR, using the algebraic equations (21-23). For the actual calculations, it is more convenient to use an inverse method. Instead of giving the values of the lumped parameter H/PrR, ζ^* is prescribed in addition to m^*, μ/μ_f, Pr_f and $\Gamma\sqrt{Pr_f}$. Since Eq. (21) can be rearranged to give u_i^* in a closed form, u_i^* may easily be evaluated from Eq. (21) for a given set of m^*, Pr_f, and ζ^*. Knowing u_i^*, Eq. (22), which may be regarded as a quadratic equation in terms of Λ, readily gives the value of Λ. Then, these results are substituted into Eq. (23) to determine H/PrR. Once these shape factors are known, the local Nusselt number $Nux=-(x/\Delta T)(\partial T/\partial y)|_{y=0}$ and the local skin-friction coefficient $C_{fx}=2\mu(\partial u/\partial y)|_{y=0}/2\rho_f u_e^2$ may be evaluated from

$$\frac{Nux}{(Rex\ i)^{1/2}}=\left(\frac{m^*}{2u_i^*\Lambda}\,\frac{\mu_f}{\mu}\right)^{1/2} \tag{25}$$

and

$$C_{fx}\left(\frac{Rex}{i}\right)^{1/2}=(1+\Lambda)\left(\frac{2m^*u_i^*}{\Lambda}\,\frac{\mu}{\mu_f}\right)^{1/2} \tag{26}$$

Results on a Flat Plate

It is obvious from Eq. (22) that Λ must vanish for $m=0$. Substitution of Eq. (22) into Eqs. (23), (25), and (26) leads to

$$\frac{H}{PrR}=\frac{15}{8}\times\frac{u_i^{*3}}{(1-u_i^*)^2[1+(3/2)u_i^*]\{1-[u_i^*/(1-u_i^*)]\zeta^*\Gamma\sqrt{Pr_f}\}} \tag{27}$$

$$\frac{Nux}{(Rex\ i)^{1/2}}\left(\frac{\mu}{\mu_f}\right)=\left[\frac{2}{15}\left(1+\frac{3}{2}u_i^*\right)\right]^{1/2}\frac{1-u_i^*}{u_i^*} \tag{28a}$$

and

$$C_{fx}\left(\frac{Rex}{i}\right)^{1/2}=\left[\frac{8}{15}\left(1+\frac{3}{2}u_i^*\right)\right]^{1/2}(1-u_i^*) \tag{28b}$$

The pair of Eqs. (27) and (21) with $m^*=0$ should be used to determine the shape factors u_i^* and ζ^* for given $\Gamma\sqrt{Pr_f}$, Pr_f and H/PrR. Such calculation results have already been reported[9] and excellent agreement with Cess and Sparrow's solutions has been confirmed. Especially, when $H/PrR\gg1$ and $\Gamma=0$, u_i^* is nearly equal to unity, hence,

$$\frac{Nux}{(Rex\ i)^{1/2}}\left(\frac{\mu}{\mu_f}\right)=\frac{1}{2}C_{fx}\left(\frac{Rex}{i}\right)^{1/2}=\frac{1}{2}\left(\frac{H}{PrR}\right)^{-1/2} \tag{29}$$

As will be shown shortly, the presence of the pressure gradient can drastically change the physical structure of the vapor flow.

Asymptotic Results in the Presence of a Pressure Gradient

For the study of the physical structure of the flow, it is most expedient to investigate asymptotic behaviors implicit in the governing equations. A careful observation on the characteristic equations (21-23) leads to various asymptotic expressions.

Results for Small Superheating ($H/PrR\ll1$)

u_i^* is naturally small for small vapor superheating. Moreover, Eq. (22) suggests that Λ must vanish with u_i^* since μ/μ_f is finite and $m\neq0$ (note, $\mu/\mu_f\cong0.06$ for the water/vapor system). Thus, Eqs. (21-23), (25), and (26) reduce to

$$u_i^*=\left(\frac{8}{15}\,\frac{1+8m^*}{1+m^*}\,\frac{H}{PrR}\right)^{1/3} \tag{30a}$$

$$\Lambda=\left[\frac{225}{8}\,\frac{m^{*3}}{(1+m^*)(1+8m^*)^2}\right]^{1/3}\frac{\mu}{\mu_f}\left(\frac{H}{PrR}\right)^{1/3} \tag{30b}$$

$$\zeta^*=\left(\frac{5}{2}\,\frac{1+m^*}{1+8m^*}\right)^{1/2}\quad\text{for } Pr_f\ll\frac{1+8m^*}{1+m^*}$$

$$\zeta^*=\left(\frac{1+m^*}{1+8m^*}\right)^{1/2}\quad\text{for } Pr_f=\frac{1+8m^*}{1+m^*}$$

$$\zeta^*=\left(\frac{5}{4}\,\frac{1+m^*}{1+8m^*}\right)^{1/3}\Big/Pr_f^{1/6}\quad\text{for } Pr_f\gg\frac{1+8m^*}{1+m^*} \tag{30c}$$

$$\frac{Nux}{(Rex\ i)^{1/2}}=\left[\frac{(1+m^*)^2(1+8m^*)}{120}\right]^{1/6}\Big/\left(\frac{\mu}{\mu_f}\right)\left(\frac{H}{PrR}\right)^{1/3} \tag{30d}$$

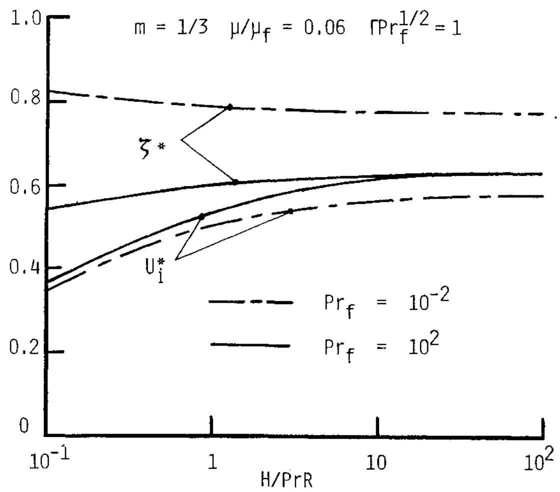

Fig. 2 Effect of H/PrR on ζ^* and u_i^*.

and

$$C_{fx}\left(\frac{Rex}{i}\right)^{1/2} = \left[\frac{8}{15}(1+8m^*)\right]^{1/2} \tag{30e}$$

Results for Large Superheating ($H/PrR \gg 1$)

Actual film boiling cases usually satisfy this condition (note, $H/PrR \cong 40\Delta T°C$ for the water-steam system.) The term $[(1-\Lambda)(1-u_i^*)]u_i^*\zeta^*\Gamma\sqrt{Pr_f}$ in the denominator of Eq. (23) representing the ratio of the heat conducted through the subcooled liquid to the total energy supplied through the wall is always positive and less than unity (note $\Lambda \gtrsim 1$ when $u_i^* \lesssim 1$). The limiting cases for $H/PrR \gg 1$ may be subdivided into the two classes, depending on whether this term is present or not, corresponding to a saturated liquid and a subcooled liquid.

Results for Saturated Liquids ($\Gamma = 0$)

Equations (22) and (23) with $\Gamma = 0$ indicate $u_i^{*2} \propto \Lambda \gg 1$ when $H/PrR \gg 1$. Thus, the set of equations reduce to

$$u_i^* = \left(\frac{75}{2}\frac{m^{*3}}{(1+m^*)(1+3m^*)^2}\right)^{1/6}\left(\frac{\mu}{\mu_f}\right)^{1/2}\left(\frac{H}{PrR}\right)^{1/6} \tag{31a}$$

$$\Lambda = \left(\frac{12}{5}\frac{1+3m^*}{1+m^*}\right)^{1/3}\left(\frac{H}{PrR}\right)^{1/3} \tag{31b}$$

$$\zeta^* = \left(\frac{5}{3}\frac{1+m^*}{1+3m^*}\right)Pr_f^{1/2} \quad \text{for } Pr_f \ll \frac{1+3m^*}{1+m^*}$$

$$\zeta^* = \left(\frac{1+m^*}{1+3m^*}\right)^{1/2} \quad \text{for } Pr_f = \frac{1+3m^*}{1+m^*}$$

$$\zeta^* = \left(\frac{5}{3}\frac{1+m^*}{1+3m^*}\right)^{1/2} \quad \text{for } Pr_f \gg \frac{1+3m^*}{1+m^*} \tag{31c}$$

$$\frac{Nux}{(Rex\ i)^{1/2}} = \left[\frac{m^*(1+m^*)}{24}\right]^{1/4} \Big/ \left(\frac{\mu}{\mu_f}\right)^{1/4}\left(\frac{H}{PrR}\right)^{1/4} \tag{31d}$$

$$C_{fx}\left(\frac{Rex}{i}\right)^{1/2} = \left(\frac{24m^{*3}}{1+m^*}\right)^{1/4}\left(\frac{\mu}{\mu_f}\right)^{3/4}\left(\frac{H}{PrR}\right)^{1/4} \tag{31e}$$

Results for Subcooled Liquids ($\Gamma \neq 0$)

The condition $H/PrR \gg 1$ can be met when the denominator of Eq. (23) becomes vanishingly small. Thus, Eq. (23) implies

$$(1-u_i^*)/u_i^* = (1-\Lambda)\zeta^*\Gamma\sqrt{Pr_f} \tag{32}$$

From Eq. (32), it is obvious that $\Lambda = 1$ for $u_i^* = 1$. Hence, Eqs. (21) and (22) reduce to

$$\zeta^* = \left(\frac{1+m^*}{1+5m^*}\right)^{1/2} \tag{33}$$

and

$$\Gamma\sqrt{Pr_f} = \left(\frac{3}{2}\frac{m^*}{1+m^*}\frac{\mu}{\mu_f}\right)^{1/2} \tag{34}$$

Two limiting cases are considered in this class, depending on whether $\Gamma\sqrt{Pr_f}$ is much greater or much smaller than that given by Eq. (34).

1) For $\Gamma\sqrt{Pr_f} \ll \left(\frac{3}{2}\frac{m^*}{1+m^*}\frac{\mu}{\mu_f}\right)^{1/2}$.

In this case, Eq. (32) together with Eq. (22) reveals $u_i^{*2} \propto \Lambda \gg 1$. In fact, the following results can be obtained from Eqs. (32), (22), (25), and (26):

$$u_i^* = \left(\frac{5}{2}\frac{m^*}{1+3m^*}\frac{1}{\zeta^*}\right)^{1/2} \Big/ \left(\frac{\mu_f}{\mu}\Gamma\sqrt{Pr_f}\right)^{1/2} \tag{35a}$$

$$\Lambda = 1/\zeta^*\sqrt{Pr_f} \tag{35b}$$

$$\zeta^* = \text{Eqs. (31c)} \tag{35c}$$

$$\frac{Nux}{(Rex\ i)^{1/2}} = \left[\frac{m(1+3m^*)\zeta^{*3}}{10}\right]^{1/4}\left(\frac{\mu_f}{\mu}\Gamma\sqrt{Pr_f}\right)^{1/4} \tag{35d}$$

$$C_{fx}\left(\frac{Rex}{i}\right)^{1/2} = \left(\frac{10m^{*3}}{1+3m^*}\frac{1}{\zeta^{*3}}\right)^{1/4} \Big/ \left(\frac{\mu_f}{\mu}\Gamma\sqrt{Pr_f}\right)^{3/4} \tag{35e}$$

2) For $\Gamma\sqrt{Pr_f} \gg \left(\frac{3}{2}\frac{m^*}{1+m^*}\frac{\mu}{\mu_f}\right)^{1/2}$.

Similar consideration for this case of large subcooling leads to the fact, $u_i^* \propto \Lambda \ll 1$, namely,

$$u_i^* = 1/\zeta^*\Gamma\sqrt{Pr_f} \tag{36a}$$

$$\Lambda = \left(\frac{15}{4}\frac{m^*}{(1+8m^*)\zeta^*}\right) \Big/ \left(\frac{\mu_f}{\mu}\Gamma\sqrt{Pr_f}\right) \tag{36b}$$

$$\zeta^* = \text{Eqs. (30c)} \tag{36c}$$

$$\frac{Nux}{(Rex\ i)^{1/2}} = \left[\frac{2(1+8m^*)}{15}\zeta^{*2}\right]^{1/2}\left(\frac{\mu_f}{\mu}\Gamma\sqrt{Pr_f}\right) \tag{36d}$$

$$C_{fx}(Rex/i)^{1/2} = \text{Eq. (30e)} \tag{36e}$$

Thus, all possible asymptotic expressions have been extracted from Eqs. (21-23). Walsh and Wilson[6] obtained two distinct asymptotic expressions, one of which is identical to Eq. (31d) for saturated liquids and the other corresponds to Eq. (36d) for large subcooling with ζ^* given by the first expression in Eq. (30c) for small Pr_f, namely,

$$\frac{Nux}{(Rex\ i)^{1/2}} = \left(\frac{1+m^*}{3}\right)^{1/2}\left(\frac{\mu_f}{\mu}\Gamma\sqrt{Pr_f}\right) \quad \text{for } Pr_f \ll \frac{1+8m^*}{1+m^*} \tag{37}$$

Walsh and Wilson's expression is the same as above, except that the multiplicative constant $[(1+m^*)/3]^{1/2}$ is replaced by $[(1+m^*)/\pi]^{1/2}$. No asymptotic expression for the large Pr_f range was suggested in their work.

Calculation Results and Discussions

Using Eqs. (21-26), calculations were carried out on a right-angle wedge ($m = 1/3$) with $\mu/\mu_f = 0.06$ assuming the water/steam system (note, the flow over a right-angle wedge corresponds to an axially symmetrical stagnation flow). Effects of H/PrR (vapor superheating) on the shape factors ζ^* and u_i^* (interfacial velocity) are indicated in Fig. 2. As H/PrR increases, u_i^* increases monotonously, while ζ^* either increases or decreases depending on whether Pr_f is greater or less than unity. Each shape factor approaches its asymptotic level as the vapor superheating (H/PrR) becomes significant.

Asymptotic values of ζ^*, Λ, and u_i^* are plotted in Figs. 3 for a wide range of $\Gamma\sqrt{Pr_f}$ (liquid subcooling). Results for $Pr_f = 10^{-2}$, 1, and 10^2 are plotted together to show the liquid Prandtl number effects. Curves for different Pr_f intersect at

$$\Gamma\sqrt{Pr_f} = \left(\frac{3}{2}\frac{m^*}{1+m^*}\frac{\mu}{\mu_f}\right)^{1/2} = 0.15$$

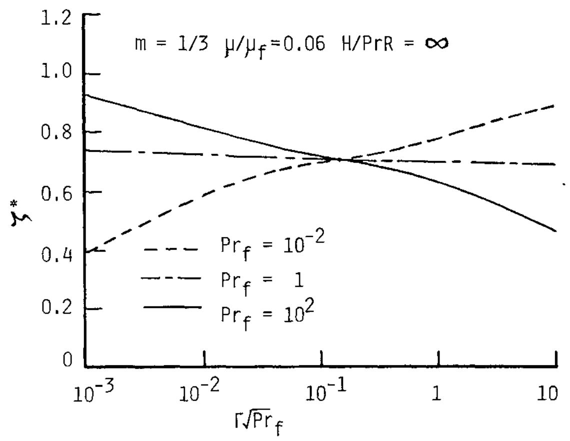

Fig. 3a Asymptotic results on ζ^*.

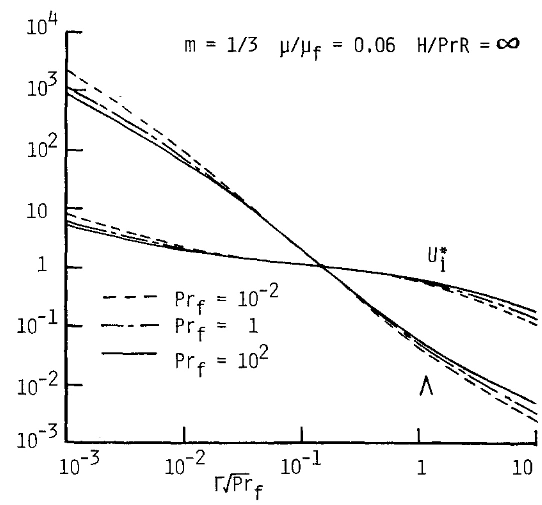

Fig. 3b Asymptotic results on Λ and u_i^*.

[see Eq. (34)]. The effect of Pr_f becomes appreciable away from this intersection point. The increase in $\Gamma\sqrt{Pr_f}$ lowers the levels of Λ and u_i^* as seen in Fig. 3b.

Heat-transfer results obtained for moderate (200°C) and high (500°C) vapor superheating are plotted in Figs. 4 with the exact solution.[6] Figure 4a also presents the case for low vapor superheating (40°C), which may become important for situations just after the onset of subcooled film boiling. The abscissa variable is set to the degree of liquid subcooling, while the ordinate variable is chosen as $(\frac{2}{3}Rex)^{1/2}/Nux$ to provide direct comparison with the exact solution. Agreement between the two solutions appears to be excellent. The fact indicates a high accuracy acquired in the present approximate method and substantiates the validity of the assumption of negligible inertia and convection terms in the vapor layer for the practical range of vapor superheating.

The heat-transfer grouping $Nux/Rex^{1/2}(\mu_f/\mu)$ and skin-friction grouping $\frac{1}{2}C_{fx}Rex^{1/2}$ are presented in Figs. 5a and 5b, respectively. The asymptotes are also indicated in these figures, following the formulas derived in the preceding section. Figure 5a clearly shows that the liquid subcooling leads to a high heat-transfer rate as a result of effective conduction through the liquid layer. The effect of Pr_f on the heat-transfer grouping is insignificant for $Pr_f \ll 1$ at large subcooling and for $Pr_f \gg 1$ at small subcooling.

Skin-friction results in Fig. 5b show a trend quite different from that observed in the case of a flat plate. It is especially interesting to note that, as H/PrR increases, the curves with small subcooling first decrease down to a minimum level,

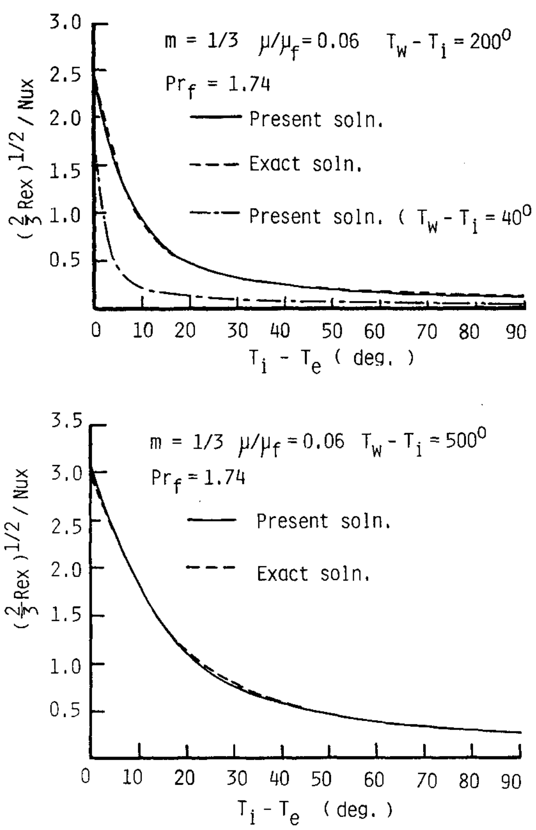

Fig. 4 Comparison of approximate solution and exact solution.

then increase and overshoot the single-phase flow level, closely following the asymptote given by Eq. (31e), and finally approach corresponding horizontal asymptotes [given by Eq. (35e)]. This fact can be explained only by considering the effect of pressure gradients. For small H/PrR (superheating), the interfacial velocity is less than the liquid freestream velocity ($u_i^* < 1$, i.e., the vapor retarding the liquid flow). Naturally, the skin friction is less for larger H/PrR since the vapor superheating tends to thicken the vapor film. However, when the vapor is superheated significantly under the negative pressure gradient, the vapor flow velocity may well exceed the liquid freesteam velocity ($u_i^* > 1$, i.e., the vapor dragging the liquid). In such a situation, an increase in H/PrR causes strong flow acceleration within the vapor film and the velocity gradient at the wall increases considerably, leading to an appreciable increase in the wall shear stress. The skin-friction grouping $C_{fx}Rex^{1/2}$ attains its minimum value when the interfacial velocity equals to the lqiuid freestream velocity (namely, $u_i^* = \Lambda = 1$). As already seen in Fig. 3b, when the liquid subcooling is sufficiently large, the interfacial velocity never attains the level of the liquid freestream velocity. Thus, the minimum appears only when

When
$$\Gamma\sqrt{Pr_f} < \left(\frac{3}{2}\frac{m^*}{1+m^*}\frac{\mu}{\mu_f}\right)^{1/2}.$$

$$\Gamma\sqrt{Pr_f} = \left(\frac{3}{2}\frac{m^*}{1+m^*}\frac{\mu}{\mu_f}\right)^{1/2},$$

it is possible to maintain a relatively low skin-friction level in the wide range of H/PrR. As $\Gamma\sqrt{Pr_f}$ increases further, the asymptotic level becomes higher, as in the case of a flat plate, and asymptotically approaches the single-phase flow level given by Eq. (30e).

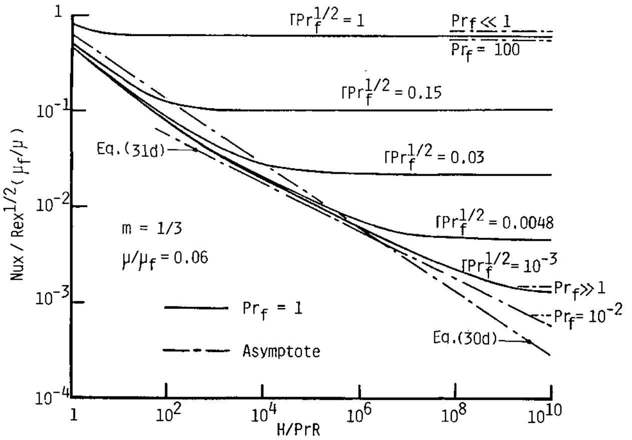

Fig. 5a Local Nusselt number.

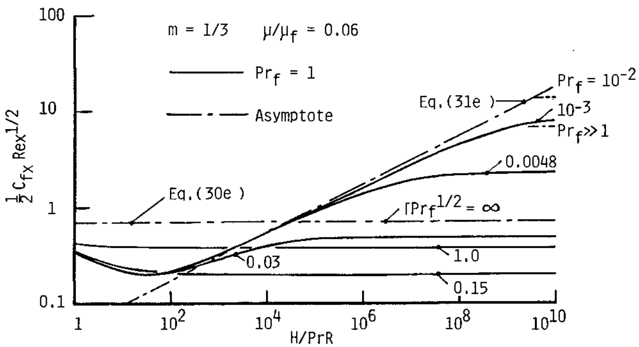

Fig. 5b Local skin-friction coefficient.

Conclusions

Upon assuming constant properties and a large liquid/vapor density ratio, forced-convection film boiling in the presence of pressure gradients has been analyzed successfully by means of an integral procedure based on the two-phase boundary-layer theory. The governing equations, along with boundary and compatibility conditions for both the vapor and liquid phases, have been reduced to a set of characteristic equations for three unknown shape factors, which can readily be solved once a set of parameters is provided. Agreement of the present approximate solution with exact solutions turns out to be excellent.

Moreover, it has been confirmed that the presence of pressure gradients changes the skin-friction and heat-transfer characteristics markedly. (In this sense, the flat-plate solution should be regarded as somewhat overidealized.) It is especially interesting to note that an excessive vapor superheating can result in significant increase in the drag force.

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